Normal Maths Mechanics Notes

Modelling in Mechanics

Model	Modelling assumptions
Particle - negligible dimensions	• mass concentrated at a single point • rotational forces and air resistance
Rod - all dimensions but one are negligible	• mass concentrated along a line • no thickness • rigid (doesn't bend, buckle, extend)
Lamina - area but negligible thickness	• mass concentrated along a flat surface
Uniform body - mass distributed evenly	• mass concentrated at geometrical centre
Light - mass is small	• treat object as being zero mass • tension the same at both ends of a light spring
Inextensible - doesn't stretch under load	• acceleration is the same in objects connected by a taut inextensible string
Smooth	• assume no friction
Rough - not smooth = rough	• objects in contact with the surface experience a frictional force
Wire	• RIGID and one-dimensional = same as rod
Smooth and light pulley - (all pulleys)	• pulley has no mass • Tension is the same either side of the pulley
Bead	• Moves freely along string / wire
Peg - A support from which a body can rest	• dimensionless and fixed • can be rough or smooth
Air resistance - friction with air	• usually negligible
Gravity	• Assume all objects attracted towards the Earth • Earth's gravity is uniformly downwards • g is constant and is $9.8\,ms^{-2}$ unless otherwise

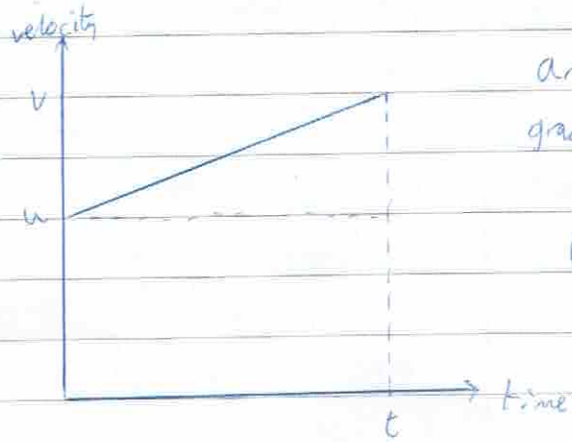
Constant Acceleration

Velocity is the rate of change of displacement. On a displacement-time graph the gradient represents the velocity (straight line = constant velocity).

average velocity = $\frac{\text{displacement from start point}}{\text{time taken}}$ average speed = $\frac{\text{total distance travelled}}{\text{time taken}}$

Acceleration is the rate of change of velocity. Velocity-time graph = gradient is the acceleration (straight line = constant acceleration), area = distance travelled.

$v = u + at$
$s = \frac{1}{2}(u+v)t$
$s = ut + \frac{1}{2}at^2$
$s = vt - \frac{1}{2}at^2$
$v^2 = u^2 + 2as$



area = $s = \frac{1}{2}(u+v)t$ ①

gradient = $a = \frac{v-u}{t}$ ②

$v = u + at$

② into ①: $s = \frac{1}{2}(u + u + at)t$

$s = \frac{1}{2}(2u + at)t$

$s = ut + \frac{1}{2}at^2$ ③

② into ①: $s = \frac{1}{2}(v - at + v)t$

$s = \frac{1}{2}(2v - at)t$

$s = vt - \frac{1}{2}at^2$ ④

② into ①: $s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$

$2as = -u^2 + v^2$

$v^2 = u^2 + 2as$ ④

The force of gravity causes all objects to accelerate towards the Earth. This acceleration is constant (ignoring air resistance) = $9.8ms^{-2}$

Forces and Motion

Newton's First: an object at rest will stay at rest and an object moving with a constant velocity will continue to move at this velocity unless an unbalanced force acts.

A resultant force acting on an object will cause acceleration in the same direction.

Two or more forces can be summed to find the resultant using vector addition.

Newton's Second: The force needed to accelerate a partical is equal to the product of its mass and the acceleration $\Sigma F = ma$ Weight = mg

Can solve problems involving connected particles by considering them separately or as a single particle.

Newton's Third: For every action there is an equal and opposite reaction.

Variable acceleration

If the displacement, s, is expressed as a function of t, then the velocity, v, can be expressed as $v = \frac{ds}{dt}$

If the velocity, v, is expressed as a function of t, then the acceleration, a, can be expressed as $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Differentiate ↓

$$\text{displacement} = s = \int v \cdot dt$$

$$\frac{ds}{dt} = \text{velocity} = v = \int a \cdot dt$$

$$\frac{dv}{dt} = \frac{d^2s}{dt^2} = \text{acceleration} = a$$

↑ Integrate

BOOK 2 ↓

Moments

Moment of F about P $= |F| \times d = |F| d \sin\theta$ ← angle between

The sum of the moments acting on a body is called the resultant moment. When a rigid body is in equilibrium the resultant force in any direction is 0N and the resultant moment about any point is 0Nm.

When a rigid body is on the point of tilting about a pivot, the reaction at any other support (or the tension in any other wire or string) is zero.

Forces and Friction

If a force is applied at an angle to the direction of motion, you can resolve it to find the component of the force that acts in the direction of motion.

SUPER USEFUL { The component of anything (eg. force) in a certain direction is $F\cos\theta$ where θ is the size of the angle between the force and the direction.

To solve problems involving inclined planes, it is easier to resolve parallel to and at right angles to the plane.

The maximum or limiting value of the friction between two surfaces, F_{max}, is $F_{MAX} = \mu R$ (μ = coefficient of friction, R = normal reaction).

Projectiles

The horizontal motion of a projectile is modelled as having constant velocity due to gravity $s = vt$

The vertical motion of a projectile is modelled as having constant acceleration due to gravity $a = g$

When a particle is projected with initial velocity U, at an angle α above the horizontal:
- The vertical component is $U\sin\alpha$
- The horizontal component is $U\cos\alpha$

A projectile reaches its point of greatest height when the vertical component of its velocity is zero.

For a particle which is projected from a point on a horizontal plane with an initial velocity U at an angle α above the horizontal, and that moves freely under gravity:

$$\text{Time of flight} = \frac{2U\sin\alpha}{g}$$

y = vertical height

x = distance horizontally from start

$$\text{Time to max. height} = \frac{U\sin\alpha}{g}$$

$$\text{Range on horizontal plane} = \frac{U^2\sin 2\alpha}{g}$$

$$\text{Equation of trajectory: } y = x\tan\alpha - \frac{gx^2(1+\tan^2\alpha)}{2U^2}$$

Applications of Forces

A particle or rigid body is in static equilibrium if it is at rest and the resultant force acting on the particle is zero. The maximum value of the frictional force F_{max} is reached when the body you are considering is on the point of moving. (This is known as limiting equilibrium). In general, the force of friction F is such that $F \leq \mu R$, and the direction of the frictional force is opposite to the direction in which the body would move if the frictional force were absent.

To be in equilibrium:
- $\Sigma F = 0$
- $\Sigma \text{moment} = 0$
- Body is stationary (for 'static' equilibrium)

Further Kinematics -- planes

If a particle starts from the point with position vector r_0 and moves with constant velocity v, then its displacement from its initial position at time t is vt and it position vector r is given by $r = r_0 + vt$ $\underline{r} = \underline{r_0} + \underline{v}t$

For an object moving in a plane with constant acceleration:

$\bar{v} = \bar{u} + \bar{a}t$ u = initial velocity, a = acceleration

$\Delta\bar{r} = \bar{u}t + \frac{1}{2}\bar{a}t^2$ v = velocity at time t, r = the displacement at t

If $r = xi + yj$, $\bar{v} = \frac{dr}{dt} = \dot{r} = \dot{x}i + \dot{y}j$ $\bar{v} = \int \bar{a} \cdot dt$

$\bar{r} = \int \bar{v} \cdot dt$

$\bar{a} = \frac{dv}{dt} = \frac{d^2r}{dt^2} = \ddot{r} = \ddot{x}i + \ddot{y}j$

constant velocity:	$\underline{r} = \underline{r_0} + \underline{v}t$
constant acceleration	$\Delta\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

⇒ $\underline{r} = \underline{r_0} + \underline{u}t + \frac{1}{2}\underline{a}t^2$